Magnetoplasmons and SU(4) symmetry in graphene

Andrea M Fischer[1], **Rudolf A Römer**[1] **and Alexander B Dzyubenko**[2,3]

[1] Department of Physics and Centre for Scientific Computing, University of Warwick, Coventry, CV4 7AL, UK

[2] Department of Physics, California State University - Bakersfield, Bakersfield, California 93311, USA

[3] General Physics Institute, Russian Academy of Sciences, Moscow 119991, Russia

E-mail: A.M.Fischer@warwick.ac.uk

Abstract. We study magnetoplasmons or neutral collective excitations of graphene in a strong perpendicular magnetic field, which can be modelled as bound electron-hole pairs. The SU(4) symmetry of graphene arising from spin and valley pseudospin degrees of freedom is explored using Young diagrams to correctly predict the degeneracies of these excitations. The multiplet structure of the states is identical to that of mesons composed of first and second generation quarks.

1. Introduction

Graphene is a single layer of graphite or set of carbon atoms sp^2 bonded together to form a honeycomb lattice. The electron in the remaining p_z orbital is delocalised and responsible for graphene being an excellent electrical conductor [1]. Graphene is associated with numerous other superlatives such as strongest material [2] and of course thinnest material; it is the first example of an atomically thick membrane. Such properties make it an excellent candidate for industrial applications, although in many cases there are obstacles to be overcome before this can be put into practice [3]. Aside from its potential usefulness, graphene is scientifically interesting in its own right. Unlike other materials, including the 2D electron gas (2DEG), electrons in graphene have a linear dispersion relation close to the Fermi energy and in this sense behave quasirelativistically [4]. Effects such as Klein tunnelling, which were previously the dominion of high-energy particle physics, have now been observed in graphene [5]. For an undoped system, the Fermi energy is located at zero energy. The Fermi surface consists of six points which coincide with the six corners of the hexagonal Brillouin zone. Only two of these points are inequivalent and are commonly termed the $\mathbf{K}$ and $\mathbf{K}'$ valleys. They may also be referred to as pseudospins, since they result in the single particle wavefunctions having a spinor form [1].

In these proceedings we examine a monolayer of graphene in the presence of a strong perpendicular magnetic field of magnitude B. The Landau level (LL) energies for graphene are, $\epsilon_n = \mathrm{sign}(n)\hbar v_\mathrm{F}\sqrt{2|n|}/\ell_B$, where $v_\mathrm{F} \sim 10^6\mathrm{ms}^{-1}$ is the Fermi velocity, $\ell_B = \sqrt{\hbar c/eB}$ is the magnetic length and $\mathrm{sign}(0) = 0$. There is also the usual LL degeneracy resulting from the different possible guiding centres for electronic cyclotron orbits. This is described by the oscillator quantum number, m, in the symmetric gauge. We consider magnetoplasmons (MPs), which are essentially excitons created from a ground state described by the filling factor $\nu = 2$, where all states with LL indices $n \leq 0$ are occupied and all other states are empty. An exciton

is formed when one of the electrons in this ground state is excited to a state in a higher lying LL and becomes bound to the hole it leaves behind. We focus on excitons with the lowest possible excitation energies, namely those with the hole in the $n = 0$ LL and the electron in the $n = 1$ LL. Taking into account the spin/pseudospin states of the electron and hole, this yields 16 resonant excitations. These are collective excitations on account of the LL degeneracy and also due to the fact that excitons with different spin/pseudospin characters may be mixed by the two-body Coulomb interaction. In the following, we study the SU(4) symmetry in graphene, which arises from the two spin and two pseudospin degrees of freedom, denoted by $\uparrow, \downarrow$ and $\Uparrow, \Downarrow$ respectively. This leads to a direct analogy between MPs in graphene and mesons composed of either first or second generation quarks. We then review the dispersion relation for MPs in pristine graphene and finally examine the states which become localised in the presence of a single Coulomb impurity. In both cases we observe the degeneracies predicted by the SU(4) symmetry and Young diagram techniques.

2. SU(4) symmetry in graphene

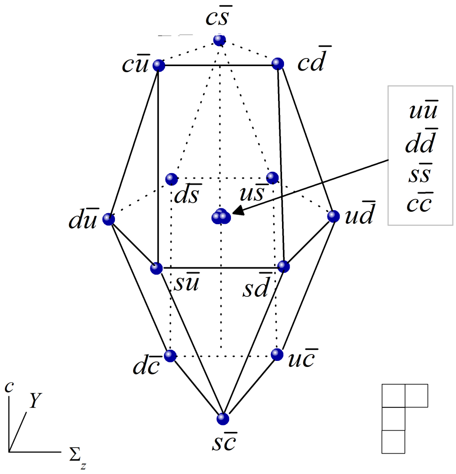

Figure 1. SU(4) [15]-plet describing the excitonic states in terms of flavour isospin (Σ_z), hypercharge (Y) and charm (c).

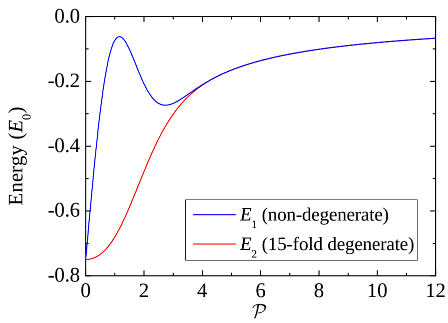

Figure 2. Dispersion relations for excitons with the hole in $n = 0$ LL and electron in $n = 1$ LL. $\mathcal{P}$ is the quasimomentum in units of $\hbar/\ell_B$.

An electron (or hole) in graphene in a strong perpendicular magnetic field is described by four quantum numbers: the LL index n; the oscillator quantum number m; the spin projection s_z and the pseudospin projection τ_z. We denote the fermionic creation operator for an electron by $c^\dagger_{nm\tau_z s_z}$ and that for a hole by $d^\dagger_{nm\tau_z s_z} \equiv c_{nm\tau_z s_z}$. The first two orbital quantum numbers are associated with the cyclotron orbits. The last two, when considered together, give four possible spin/pseudospin "flavours", leading to the SU(4) symmetry. We identify these with the four quark flavours up (u), down (d), charm (c) and strange (s) according to $\{\Downarrow\downarrow, \Uparrow\downarrow, \Downarrow\uparrow, \Uparrow\uparrow\} \equiv \{d, u, s, c\}$. The generators of SU(4) can be constructed using bilinear combinations of the fermionic creation and annihilation operators conserving the number of electrons and holes. We have

$$\hat{C}_{ij} = \sum_{nm} c^\dagger_{nmi} c_{nmj} - \sum_{nm} d^\dagger_{nmj} d_{nmi}, \tag{1}$$

where $i, j \in \{d, u, s, c\}$. The generators satisfy the commutation relations

$$[\hat{C}_{ij}, \hat{C}_{kl}] = \delta_{jk}\hat{C}_{il} - \delta_{il}\hat{C}_{kj}, \tag{2}$$

so are closed under commutation, confirming that they form a Lie algebra. The operators associated with spin and pseudospin may of course be expressed in terms of the generators. For example, $S_z = \frac{\hbar}{2}\left(\hat{C}_{ss} + \hat{C}_{cc} - \hat{C}_{uu} - \hat{C}_{dd}\right)$ and $S_+ = S_x + iS_y = \hbar\left(\hat{C}_{cu} + \hat{C}_{sd}\right)$.

In particle physics, mesons, which are a bound quark-antiquark pair, are classified using Young diagrams [6]. We use the same techniques to determine the multiplet structure for electron-hole (e-h) complexes in graphene. Young diagrams describe SU(n) multiplets and consist of n or fewer left-justified rows of boxes, such that each row is as least as long as the row beneath it. Roughly speaking, each box represents a particle. Each SU(n) multiplet is uniquely labelled by a list of $n-1$ non-negative integers, $(a_1, a_2, \ldots, a_{n-1})$. The integer in the i^{th} position corresponds to the number of boxes in the i^{th} row minus the number of boxes in the $(i+1)^{\text{th}}$ row i.e. the overhang of the i^{th} row. By the definition of a multiplet label, adding columns of length n to the left of a Young diagram is redundant. Young diagrams can be used to calculate the multiplicity, $[N(n)]$, of an SU(n) multiplet. For the $n = 4$ case of current interest,

$$[N(4)] = \frac{(a_1+1)}{1}\frac{(a_2+1)}{1}\frac{(a_3+1)}{1}\frac{(a_1+a_2+2)}{2}\frac{(a_2+a_3+2)}{2}\frac{(a_1+a_2+a_3+3)}{3}. \tag{3}$$

There are two fundamental representations of SU(n): $[n]$, with Young diagram $\square$ and label $(1, 0, \ldots 0)$ and $[\bar{n}]$ with Young diagram composed of $n-1$ vertically stacked boxes and label $(0\ldots, 0, 1)$. The latter represents an antiparticle; this makes sense if we identify n vertically stacked boxes with the vacuum. We shall also use the symbol, $\underline{1}$ to denote the vacuum. For the current purpose, the hole in a MP behaves as an antiparticle. Thus the electron has Young diagram $\square$ and the hole Young diagram $\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}$. To determine the SU(4) multiplet structure for the MPs, we must combine these two diagrams. The rules for combining Young diagrams [6] yield

$$\begin{array}{|c|}\hline \\ \hline \end{array} \otimes \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array} = \begin{array}{|c|c|}\hline & \\ \hline \\ \cline{1-1} \\ \cline{1-1} \end{array} \oplus \underline{1}. \tag{4}$$

Eq. (4) is equivalent to $(1, 0, 0) \otimes (0, 0, 1) = (1, 0, 1) \oplus (0, 0, 0)$ or $[4] \otimes [\bar{4}] = [15] \oplus [1]$. Now the generators, $\hat{C}_{ij}$, commute with the interacting Hamiltonian describing the electron system up to some small symmetry breaking terms [7]. In addition, the application of a generator to a state in a particular multiplet always returns a state in that same multiplet with the same Coulomb interaction energy. This ensures that states within a given multiplet are degenerate in their energies. Thus from this simple calculation, we have shown that the MPs in graphene will either have degeneracy 1 or 15. The [15]-plet is shown in Fig. 1 and is analogous to the meson multiplet of particle physics. The states are plotted as a function of: the flavour isospin projection $\hat{\Sigma}_z = \frac{1}{2}\left(\hat{C}_{uu} - \hat{C}_{dd}\right)$, the hypercharge $\hat{Y} = \frac{1}{3}\left(\hat{C}_{uu} + \hat{C}_{dd} + \hat{C}_{cc}\right) - \frac{2}{3}\hat{C}_{ss}$ and the charm $\hat{c} = \hat{C}_{cc}$. One may deduce the symmetry of states under exchange of quantum numbers for different particles from their Young diagrams [6].

3. Dispersion relation for magnetoplasmons in pristine graphene

As a precursor to our results for localised MPs in dirty graphene, we discuss the dispersion relation for extended MPs in a clean system. The calculation is explained in Ref. [8] in the same spirit as that for the 2DEG established by Kallin and Halperin [9], but with the additional

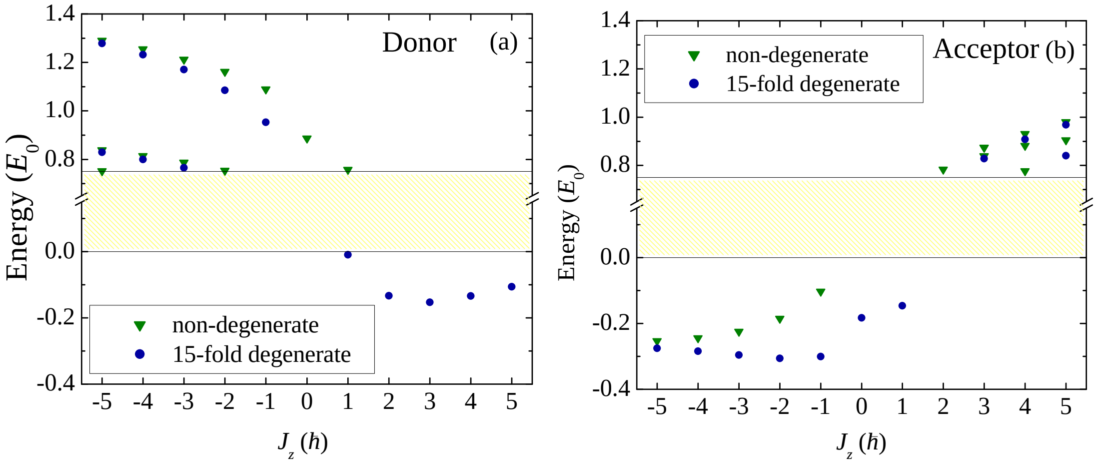

Figure 3. MPs from the ground state $\nu = 2$ localised on (a) a donor impurity and (b) an acceptor impurity. The hatched area of width $0.75E_0$ represents the continuum of extended MPs. Energies are given relative to the lower continuum edge in units of E_0 as a function of the generalised angular momentum projection, J_z.

complication of the valley pseudospin degree of freedom. The centre of mass quasimomentum for the exciton, $\mathbf{P}$, can be shown to be a good quantum number. One can think of this as being due to the repulsive Lorentz forces and attractive Coulomb forces, which act on the electron and hole, cancelling out when they have a certain separation, enabling the exciton's centre of mass to move in a straight line. The Hamiltonian (in the symmetric gauge) for MPs is [10]:

$$\begin{aligned}\hat{H} &= \sum_{\mathcal{N},m} \tilde{\epsilon}_n c^\dagger_{\mathcal{N}m} c_{\mathcal{N}m} - \sum_{\mathcal{N},m} \tilde{\epsilon}_n d^\dagger_{\mathcal{N}m} d_{\mathcal{N}m} \\ &+ \sum_{\substack{\mathcal{N}_1,\mathcal{N}_2\\ \mathcal{N}'_1,\mathcal{N}'_2}} \sum_{\substack{m_1,m_2\\ m'_1,m'_2}} \mathcal{W}^{\mathcal{N}'_1 m'_1 \mathcal{N}'_2 m'_2}_{\mathcal{N}_1 m_1 \mathcal{N}_2 m_2} c^\dagger_{\mathcal{N}'_1 m'_1} d^\dagger_{\mathcal{N}'_2 m'_2} d_{\mathcal{N}_2 m_2} c_{\mathcal{N}_1 m_1},\end{aligned} \tag{5}$$

where we define the collective index $\mathcal{N} = \{n\tau_z s_z\}$. The first two terms give the single particle contribution. The $\tilde{\epsilon}_n$ are the LL energies renormalised by a self energy correction due to exchange interactions. The last term describes the e-h interaction; it has direct and exchange components. This Hamiltonian mixes the four excitons for which there are no spin or pseudospin flips and leaves the remaining twelve unmixed. The Coulomb interaction is treated exactly in the span of given LLs and the dispersion relations are found by diagonalising the Coulomb interaction Hamiltonian in the basis of two-particle electron-hole states compatible with magnetic translations (having definite quasimomentum P). The Coulomb interaction is treated as a perturbation and the dispersion relation found by diagonalising the Coulomb interaction matrix in the basis of these non-interacting two-particle wavefunctions. For the fully filled LL considered here, there are only two possible dispersion relations. The four mixed excitons give a non-degenerate dispersion branch [8]

$$E_1 = -\frac{E_0}{8} e^{-\frac{\mathcal{P}^2}{4}} \left[\left(6 + \mathcal{P}^2\right) I_0 \left(\frac{\mathcal{P}^2}{4}\right) - \mathcal{P}^2 I_1 \left(\frac{\mathcal{P}^2}{4}\right) \right] \tag{6}$$

and a triply degenerate dispersion branch

$$E_2 = E_1 + E_0 \mathcal{P} e^{-\frac{\mathcal{P}^2}{2}}, \tag{7}$$

where $\mathcal{P} = P\ell_B/\hbar$ is the dimensionless centre of mass quasimomentum, P, for the exciton and I_0 and I_1 are modified Bessel functions. The dispersion, E_2, is the same as for tthe 12 unmixed excitons, making the total degeneracy 15, as predicted in the previous section. These dispersion relations are plotted in Fig. 2. Energies are given in units of $E_0 = \sqrt{\frac{\pi}{2}}\frac{e^2}{\varepsilon\ell_B}$, the characteristic Coulomb interaction energy, where ε is an effective dielectric constant.

4. Localised magnetoplasmons in dirty graphene

We now consider how MPs may become localised in the presence of a single Coulomb impurity with potential, $V(r) = \frac{Ze^2}{\varepsilon r}$, where Z is the impurity charge in units of e [10]. The centre of mass quasimomentum is no longer well defined for a MP. However, the axial symmetry remains and the orbital angular momentum projection, defined in terms of the electron and hole orbital quantum numbers as, $J_z = |n_e| - |n_h| - m_e + m_h$, is a good quantum number. In the presence of an impurity, localised MPs break away from the continuum of extended MPs as discrete states. These localised MPs are shown for a donor ($Z = 1$) and acceptor ($Z = -1$) impurity in Fig. 3, plotted as a function of J_z. The yellow shaded area represents the continuum; it has width $0.75E_0$, as can be seen in Fig. 2. Note that for both impurity cases, there are non-degenerate branches and branches with 15-fold degeneracy. The predictions based on SU(4) symmetry hold here, because the Coulomb impurity, being long ranged, does not introduce an inequivalency between the valleys. This would not be the case for a short-ranged impurity located on one of the carbon sites [11].

5. Conclusions

In conclusion, we have studied neutral collective excitations of graphene in the presence of a strong perpendicular magnetic field. Electrons and holes in graphene can exist in four possible spin/pseudospin states, so that electron-hole complexes may be likened to mesons composed of quarks with four possible flavours. This analogy enabled us to use techniques established for treating mesons to obtain the multiplet structure of the neutral collective excitations for both extended excitations and those localised on a Coulomb impurity. An extension of this idea to charged collective excitations, which can be thought of as three particle complexes, can be found in Ref. [12]. These are analogous to baryons formed of three quarks.

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